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THE METHOD OF LINES FOR THE ANALYSIS OF PLANAR WAVEGUIDES HAVING UNIAXIALLY ANISOTROPIC SUBSTRATES

B. M. Sherrill and N. G. Alexópoulos

Electrical Engineering Department, University of California at Los Angeles,
Los Angeles,California, 90024

Abstract

The efficient Method of Lines is formulated for the dispersive characterization of planar waveguide structures having uniaxially anisotropic dielectric substrates. This new formulation is exercised upon a configuration of fin-line, one with a substrate bearing an isolated strip opposite a fin. Numerical results follow displaying the importance of heeding anisotropy in modeling.

Introduction

The pioneering application of the Method of Lines (MOL) to microwave work by Schulz, et. al [1],[2] affords an accurate, full wave model of isotropic waveguide structures consisting of plane dielectric regions separated by interface metallization. The course of this paper specializes this scheme to an uniaxial case. Under MOL all spatial variables but one of a scalar wave equation system are discretized to effect a considerably simpler system. No infinite summations, integrals, nor basis functions are present to impede computation resulting in an economical method. This new formulation is applied towards an interesting, relatively unstudied form of fin-line. Comparisons of our results to published ones for it are excellent. Subsequently this paper reports the first anisotropic studies of the structure. These deviate significantly from isotropic ones.

Mathematical Formulation

The considered geometry is that of Figure 1 (The two air regions can be dielectric, if desired.). This 'fin-strip' is a uniform line possessing a fin of spacing s opposite a strip of width w on a dielectric substrate of thickness t. Conductors are assumed to be negligibly thin and lossless. This formulation

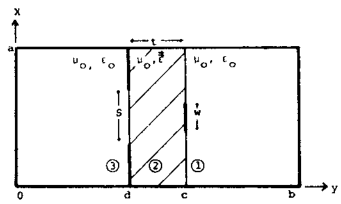

Figure 1 -Fin-Line/Strip Configuration

assumes uniaxial regions with optical axes normal to region interfaces. Though other choices are possible, this selection is worthy due to the frequency in which material of this orientation is supplied. Hence, each region possesses a tensor permittivity of the form

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_\perp \end{bmatrix} \tag{1}$$

Seeking z-propagating field solutions through potentials leads to the scalar wave equations

$$\frac{\partial^2}{\partial x^2}\Psi^e(x,y) + n^2\frac{\partial^2}{\partial y^2}\Psi^e(x,y) + (k_\parallel^2 - \beta^2)\Psi^e(x,y) = 0 \tag{2.a}$$

and

$$\frac{\partial^2}{\partial x^2}\Psi^h(x,y) + \frac{\partial^2}{\partial y^2}\Psi^h(x,y) + (k_\perp^2 - \beta^2)\Psi^h(x,y) = 0 \tag{2.b}$$

where
$k_\parallel = n_\parallel k_0$, $k_\perp = n_\perp k_0$, $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, $n_\parallel = \sqrt{\varepsilon_\parallel}$, $n_\perp = \sqrt{\varepsilon_\perp}$, $n = n_\parallel/n_\perp$. For later computational examples the usual case of conductors placed symmetrically upon the substrate is assumed. Considering half the guide cross section with a magnetic wall then yields an equivalent problem still possessing the dominant mode. Thus for the cross section of Figure 2, MOL discretizes the potentials into N interior functions each $\Psi_i^{e,h}(y) = \Psi^{e,h}(x_i^{e,h}, y)$, $i = 1, \ldots, N$ with spacing $h = (x_{N+1}^e - x_0^h)/(N + 1/2)$. This discretization reduces the gov-

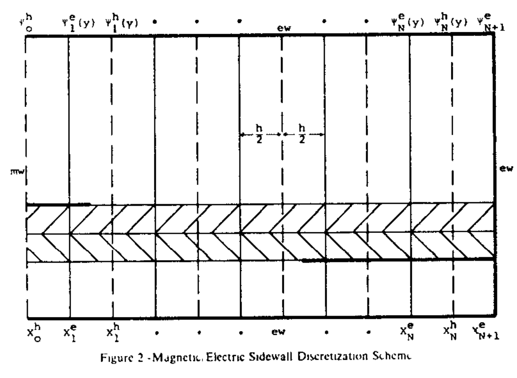

Figure 2 -Magnetic, Electric Sidewall Discretization Scheme

erning partial differential equation (PDE) system to an ordinary (ODE) one whose coupled nature can be undone by a simple, analytic, linear transformation [2]. Solutions to this discretized system are simply (for i = 1,...,N)

$$\Phi_i^e(y) = A_i^e \cosh\left(\frac{\kappa_{ei} y}{nh}\right) + B_i^e \sinh\left(\frac{\kappa_{ei} y}{nh}\right) \tag{3.a}$$

and

$$\Phi_i^h(y) = A_i^h \cosh\left(\frac{\kappa_{hi} y}{h}\right) + B_i^h \sinh\left(\frac{\kappa_{hi} y}{h}\right) \tag{3.b}$$

where κ_{ei}, κ_{hi} are discretized wavenumbers related to permittivities and $A_i^{e,h}, B_i^{e,h}$ are potential coefficients indirectly solved upon application of boundary conditions. In this 'transformed' domain all matrices are diagonal and so behave as vectors. MOL derives computational speed from this, since operations may be effected element-by-element rather than by grand manipulation. The aforementioned enforcement culminates in an interface equation relating interface fields to interface currents

$$\begin{bmatrix} \overline{E}_{z2c} \\ \overline{E}_{x2c} \\ \overline{K}_{zd} \\ \overline{K}_{xd} \end{bmatrix} = [\,\overline{\overline{S}}\,] \begin{bmatrix} \overline{K}_{zc} \\ \overline{K}_{xc} \\ \overline{E}_{z2d} \\ \overline{E}_{x2d} \end{bmatrix} . \tag{4}$$

The quantities $\overline{E}$ and $\overline{K}$ are the discretized electric field and sheet current density with the subscripts c,d and 2 indicating evaluation at y = c,d in region 2. Thus, the interface boundary condition for the strip and slot

$$\begin{bmatrix} \overline{E}_{z2c} \\ \overline{E}_{x2c} \\ \overline{K}_{zd} \\ \overline{K}_{xd} \end{bmatrix}_{red} = \overline{0} \tag{5}$$

at last implies the problem's dispersion equation

$$\det\left[\,\overline{\overline{S}}(\omega, \beta)\,\right]_{red} = 0 \tag{6}$$

The mark 'red' abbreviates 'reduced' and signifies inclusion only for matrix elements associated with potential lines intersecting the strip and slot. In upcoming examples 'small' (i.e. less than half the interface length) strips and slots are chosen, and so the form of (4) further reduces computational effort (i.e. (6) is of smallest order). Hence, (6) yields upon solution all phase constants of propagating modes. Note that due to the isolated strip this structure always possesses an active mode (therefore the dominant). With β known, all guide fields are explicitly calculable allowing further computations of interest to proceed (e.g. impedance).

Computational Results

The Method of Lines developed in this paper enjoys very good agreement with other methods. Comparisons to isotropic shielded microstrip [3] and to anisotropic coplanar waveguide [4] show an average difference of 0.5% (N = 9 and 53 respectively) from these other theoretical calculations. This paper's structure for isotropic substrates [5] also compares well by displaying a 0.2% average difference (N = 27) from the alternate theory. Collectively these positive comparisons support this anisotropic MOL formulation. Passing these checks consider next the issue of anisotropy for 'finstrip' brought up in Figures 3 and 4. These curves display effective permittivity ($\varepsilon_{eff} = (\beta/k_0)^2$) and impedance (strip current-guide power) calculations to 0.5% accuracy (N = 33) for a PTFE substrate. The usual choices of symmetrically placed conductors, a symmetrically placed slab, and a standard guide (b/a = 2) are taken. PTFE denotes a common class of ceramic impregnated, low permittivity teflon materials possessing manufacture induced anisotropy. The member

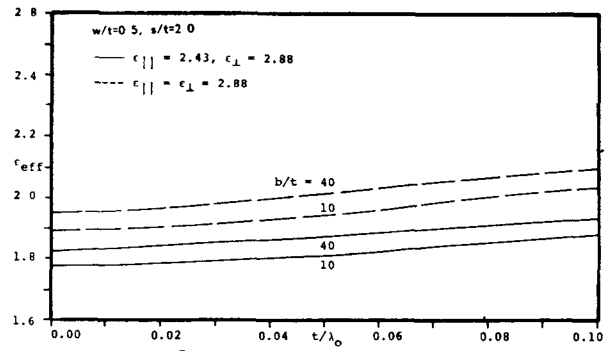

Figure 3 - Effective Permittivity for a PTFE Dielectric Case

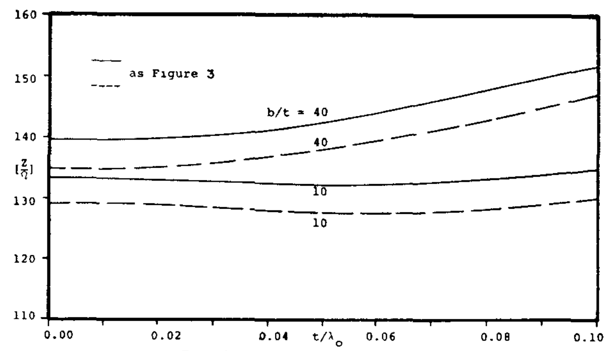

Figure 4 - Impedance for a PTFE Dielectric Case

chosen here has the values $\varepsilon_{\parallel} = 2.43, \varepsilon_{\perp} = 2.88$. The figures present curves for this and for the isotropic approximation $\varepsilon_{\parallel} = \varepsilon_{\perp} = 2.88$. This approximation leads to errors in ε_{eff} of up to +8.8% and in Z of up to -3.6% for the range shown. Compounding these typical errors with the frequent use of anisotropic substrate materials lends weight to heeding anisotropy in modeling.

Conclusion

The Method of Lines has been applied to a planar waveguide having an uniaxially anisotropic substrate for which dispersive effective dielectric constants and impedances were calculated. The results from this efficient, full wave scheme made obvious the errors that would be incurred if anisotropy were to be neglected.

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